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Original Scientific Paper

Achromatic Ebbinghaus Optical Illusion in the Printing Process

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Abstract: In the printing process, geometric optical illusions often appear, affecting the perception of shapes by observers. Under the influence of various geometric structures, visual illusions may occur that distort the perception of geometric properties as seen by the observer. This paper presents the results of a psychophysical visual experiment in which three different achromatic variants of the Ebbinghaus illusion with white and black circles were tested. The aim was to investigate how this illusion depends on the colouring of the outer and inner Ebbinghaus circles in printed materials. Samples were defined with white circles, black circles, and black inner with white outer circles. The study involved 27 participants of different genders, aged 18 to 23 years. Differences in illusion strength across samples were tested using the non-parametric Friedman ANOVA for repeated measures, followed by post-hoc analysis with Wilcoxon matched-pairs tests. The existence of pairs of samples that differ statistically significantly was established. Furthermore, a statistically significant difference was found between the samples with all white and all black circles ($p < 0.05$), while no statistically significant differences were found among the other pairs. Descriptive statistics showed that the strongest illusion intensity occurred in the samples with black-coloured inner and outer circles.

Keywords: geometric optical illusions; Ebbinghaus optical illusion; visual communication; printing process.

1. Introduction

Geometric optical illusions in the literature [1,2] are defined as psychophysical visual phenomena that cause a change in the geometric properties of objects as seen by the observer. Illusions manifest as a change in the perception of the geometric features of objects. Thus, lines, surfaces, volumes, curves, angles, and spatial relations deviate for the observer from their actual physical measures [2,3]. In other words, the observer may perceive a line as longer or shorter than its actual length, a circle as smaller or larger than its actual area, a straight line as very curved, a right angle as obtuse or acute, and an object at rest may appear to move or rotate.

Geometric optical illusions that frequently occur in graphic media are the Müller-Lyer, Ebbinghaus, Delboeuf, Ponzo, Poggendorff, Zöllner, Hering, Orbison, Kanizsa illusions, the Penrose triangle, and many other illusions [4-6]. Scientific research on geometric optical illusions in the printing process requires an interdisciplinary approach, which involves using mathematical and statistical tools, as well as results from neuroscience research in combination with methods from graphic sciences.

In the printed medium, the occurrence of geometric optical illusions arises from different combinations of colours and shapes that make up the structure of the design. Often, geometric optical illusions appear unintentionally in printing. In such cases, visual illusions that occur accidentally in printing can sometimes seriously compromise the quality and clarity of the print. However, most geometric optical illusions in printing are implemented intentionally to achieve a more attractive, modern, and dynamic design. Designers can also deliberately use visual illusions, and it is important to know their fundamental principles [7]. Recently, the understanding has begun to prevail that geometric optical illusions form an important part of the language of contemporary visual communications.

Graphic designers know that context can often influence the distorted perception of size. It is known that an object appears larger in a context of smaller objects and smaller in a context of larger objects. This perceptual law was used by the German psychologist Hermann Ebbinghaus (1850-1909), who first defined the optical illusion named after him as the Ebbinghaus optical illusion (Figure 1) [3].

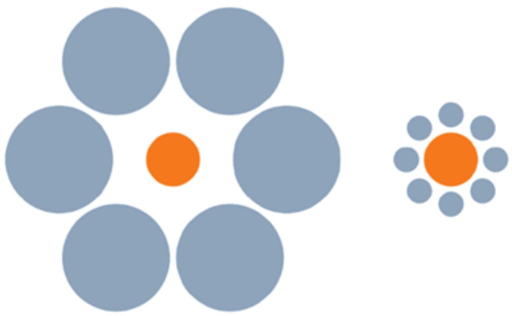

Figure 1. Ebbinghaus optical illusion.

The Ebbinghaus structure shown in Figure 1 consists of an orange central circle surrounded by large and small blue circles. The large blue circles are located on the left side, while the small orange circles are located on the right side of the figure. The central orange circle, which is between the surrounding circles, is physically identical on both the left and right sides. However, to most observers, it appears that the left central circle is much smaller than the right central circle. The reason for the illusion is the structure around the central circle, where the central circle surrounded by large circles appears smaller, and the one surrounded by smaller circles appears larger.

The illusion effect does not occur randomly but has been scientifically confirmed in numerous neuroscience studies using statistical methods and models, Gaussian and Gabor filters, Fourier analysis, differential operators, neural networks, and many other mathematical methods. An experiment was conducted, and using selected statistical tests, it was shown that the strength of the illusion depends on the number and arrangement of circles, which means that this illusion is caused not only by size contrast but also by the interaction of the inner circle with the outer circles [8]. Visual neuroscience research has explained the neural mechanisms in the brain that cause the appearance of the Ebbinghaus illusion [9]. It has been shown that the illusion is predominantly monocular, meaning that its intensity is greater when viewed with only one eye. This indicates that the illusion is generated in the early visual cortex, as the visual system produces the illusion without information from both eyes. The result was obtained using a linear regression model with independent variables describing the effect of binocular, monocular, and dichoptic conditions, using linear and nonlinear interactions of variables.

There are attempts to interpret the Ebbinghaus illusion using Gabor filters, which are directly related to Gabor waves, or Fourier analysis [10-12]. Recent studies have shown that this illusion is complex in that multiple parameters determine its strength [13]. In addition to the distance between the surrounding circles and the inner circle, the number of outer circles also affects the illusion. The conclusions of the studies were confirmed by ANOVA and correlation analysis. Previous work was extended with a mathematical and neurological model that explains the Ebbinghaus and Delboeuf illusions based on space deformation achieved through differential operators [14]. The model starts from the basic assumption that the shift in perception is a function of the local derivative, through which abrupt changes in colour or object contours are detected. The assumed function deforms space by reducing or expanding it. The proposed mathematical model takes the distance between the outer circles (surroundings) and the inner circle as a key parameter determining the strength of the illusion. The authors demonstrated that their neuro-mathematical model closely follows experimental data from previous research [13] and indicated the possibility of constructing similar models for other geometric optical illusions [14].

The contemporary approach to mathematically modelling geometric optical illusions, including the Ebbinghaus illusion, also involves the use of classical convolutional neural networks [15]. Such networks are constructed to mimic the functioning of the human visual cortex through a combination of filters, contours, edges, noise removal, and other parameters. A study was published showing that convolutional neural networks (CNNs) closely replicate the human visual system in the case of geometric optical illusions, even when they were not previously trained on optical illusions [16,17]. There are also successful attempts to describe geometric optical illusions using deep neural networks (DNNs) [18]. Finally, more complex recurrent convolutional neural networks (RCNNs) simulate the functioning of the visual cortex even more accurately and correspond more closely to the perception of geometric optical illusions [19].

The presented studies provide opportunities for applying mathematical models of geometric optical illusions in the printing process to identify geometric optical illusions. The identification of geometric optical illusions in printed materials can improve quality control in the printing process to avoid unwanted effects caused by these illusions. In this study, statistical methods were used to test how colouring the circles with achromatic black affects the strength of the Ebbinghaus illusion on printed materials.

2. Methodology

The experiment was planned in two parts. The first part consisted of two phases. In the first phase, test cards with the achromatic Ebbinghaus illusion were designed in Adobe Illustrator 28.2. In the second phase of the experiment, the test cards were printed on a calibrated Accurio Print C750i Professional Printer (Konica Minolta). The second part consisted of a psychophysical visual experiment in which 27 participants took part. Statistical data processing was performed using the Statistica 13 software package (Stat Soft Tools).

2.1. Design and reproduction of test cards

Three test cards, or three samples, were designed (Figure 2 a–c, Table 1). Sample 1 (Figure 2a, Table 1) was made with white inner and outer circles. Sample 2 (Figure 2b, Table 1) was made with inner and outer circles coloured black. These are filled circles. The third sample, Sample 3 (Figure 2c, Table 1), had inner circles coloured black, while the outer circles were white. The inner circle is filled, while the outer circles are unfilled. The Ebbinghaus circles are shown on the test cards. On the right side of the test cards, reference circles are placed. Two inner circles (left and right) are equal, with lengths of exactly 10 mm. The offered reference circles have diameters of a) 70%, b) 80%, c) 90%, d) 100% (equal), e) 110%, f) 120%, and g) 130% of the inner circles' diameter. Specifically, the diameters of the offered reference circles are a) 7 mm, b) 8 mm, c) 9 mm, d) 10 mm, e) 11 mm, f) 12 mm, and g) 13 mm.

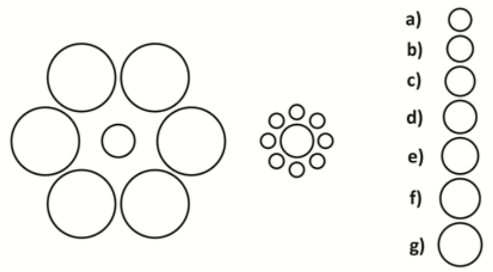

Figure 2a. Sample 1a: White outer and inner circles.

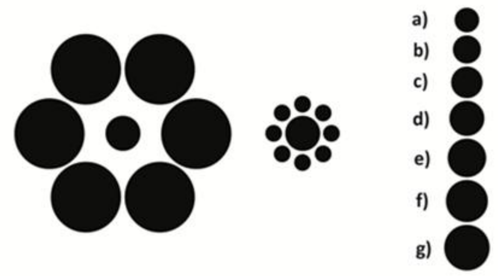

Figure 2b. Sample 2: Black outer and inner circles.

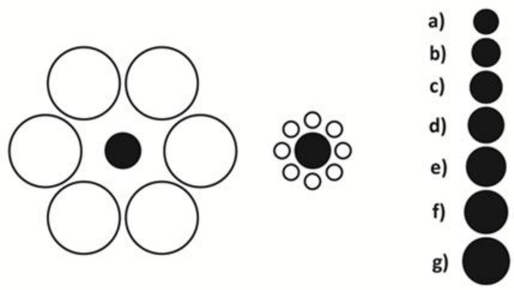

Figure 2c. Sample 3: White outer and black inner circles

The Ebbinghaus circles are defined by the following Table 1.

Then the samples were printed on a calibrated Accurio Print C750i Professional Printer (Konica Minolta). Rendering was in JPEG format in Adobe Illustrator 28.2 with CMYK colour display. ISO standard 3664:2009 defines viewing conditions for graphic technology and professional photography, and the dimensions of the test cards were adjusted accordingly. The printed substrate was high-gloss coated paper intended for laser printing, with a weight of 300 g/m^2, which was conditioned in the room for 48 hours before printing at a temperature of 23 °C and a relative humidity of 55 %.

Table 1. Ebbinghaus samples.

SAMPLES	INNER CIRCLES	OUTER CIRCLES
Sample 1	white	white
Sample 2	black	black
Sample 3	black	white

2.2. Psychophysical visual experiment

The experiment on the participants was conducted using the verified Method of Adjustment [20]. The participants were given three test cards (Figure 2 a–c). They were then assigned the task of determining the size of the left circle and the size of the right circle by comparing them with the reference circles, as they perceived them. For this purpose, a questionnaire with offered responses from a) to g), corresponding to the perceived values, was given to the participants. All participants perceived the right circle as larger than the left circle. The percentage difference in the perception of the size of the right circle (D), perceived as larger, and the left circle (L), perceived as smaller, was calculated. The diameter of the reference circle is denoted as RK in the following formula.

$$Power\ of\ illusion = \frac{D - L}{RK}$$

Data were collected from all participants, and a statistical analysis of the data was performed.

3. Results and discussion

3.1. Descriptive statistical analysis

The descriptive statistical analysis, which includes means and medians as measures of the strength of the Ebbinghaus illusion, is shown in the following Table 2 and in the Box & Whisker diagram (Figure 3).

Table 2. Descriptive statistical analysis of the Ebbinghaus illusion on achromatic samples.

SAMPLES	Descriptive Statistics			
	Mean±Std.Dev.	**Med.**	**Min-Max**	**Var.**
Sample 1: White circles	13.33±6.20	10	0-30	38.46
Sample 2: Black circles	18.15±10.75	20	0-40	115.67
Sample 3: White outer and black inner circles	15.93±9.71	10	0-40	94.30

Legend: *The table contains means (Mean), medians (Med), minimums and maximums (Min and Max), variances and standard deviations (Var, Std. Dev).*

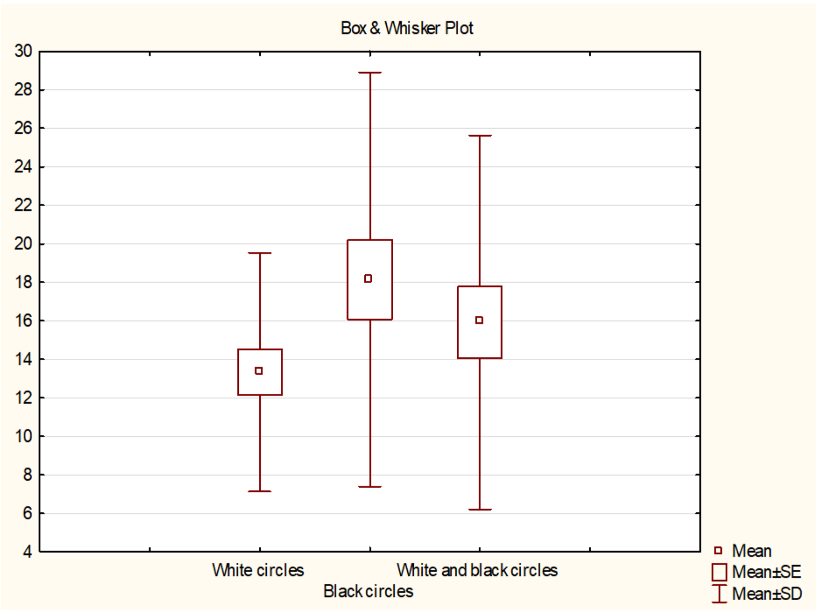

Figure 3. Box & Whisker diagram.

The descriptive statistical analysis clearly shows how the colouring of the surfaces affects the strength of the illusion. The illusion is most strongly manifested in Sample 2 with black-coloured outer and inner circles, with a mean of Mean = 18.15% and a median of Med = 20% (Table 2, Figure 3). This means that the expected shift in the perception of the size difference between the right and left circles is very large. In the mixed Sample 2, with a black inner circle and uncoloured outer circles, the strength of the illusion has a mean of Mean = 15.93% and a median of Med = 10%. The Ebbinghaus illusion is weakest in Sample 1, with white inner and outer circles, with a mean of Mean = 13.33% and a median of Med = 10%. The illusion effect is very strong in all samples, so the ranges between the minimum and maximum varied from 0% to 40% in Samples 2 and 3 (coloured and partially coloured), while in Sample 1 variations of 0% to 30% were observed (Table 2). Considering the variability in the perception of deviations from the physical value of the circles due to the illusion, variances were determined as Var = 94.30 for Sample 3, and a high variance of Var = 115.67 for Sample 2. Sample 1 had a smaller variance of Var = 38.46 (Table 2). Corresponding standard deviations were observed as well (Table 2, Figure 3).

The frequency table provides an overview of the responses by participants (Table 3). The data on frequencies from Table 3 are clearly shown in the following Figure 4.

Table 3. Frequency table of the achromatic Ebbinghaus illusion.

Category	Sample 1: White circles		Sample 2: Black circles		Sample 3: White outer and black inner circles	
	No.	Perc.	No.	Perc.	No.	Perc.
0%	1	3.70%	1	3.70%	1	3.70%
10%	17	62.96%	12	44.44%	16	59.26%
20%	8	29.63%	8	29.63%	4	14.81%
30%	1	3.70%	3	11.11%	5	18.52%
40%	0	0.00%	3	11.11%	1	3.70%

Legend: *The table contains numerical frequencies of responses (No.) and relative frequencies in percentages (Perc).*

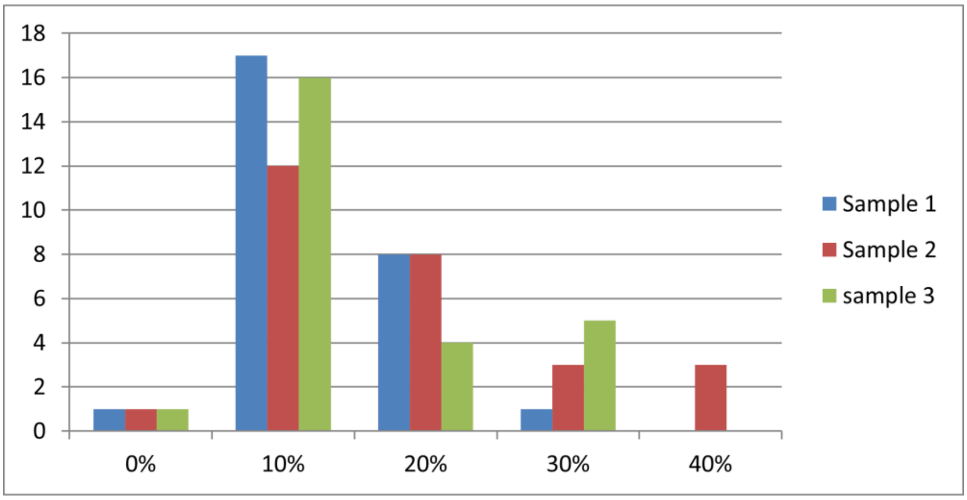

Figure 4. Distribution of frequencies of the visual experiment results.

From Table 3 (Figure 4), it can be seen that the highest frequency for all samples is at the strength of a 10% shift (No = 17, No = 12, No = 16). In Sample 1 with white circles, the frequency of the response category of 10% is No = 17, which corresponds to Perc = 62,96 of participants. No participant in Sample 1 noticed a difference of 40% (No = 0). In the sample with black circles, 3 participants (No = 3) perceived a difference of 30% and 3 participants perceived a difference of 40% (No = 3), which gives a total of 22.22% chance that the illusion effect will be greater than the very large 30% (Table 3). The illusion is also very strong in Sample 3, in which 6 participants, or 22.22% of all participants, perceive the illusion as stronger than a 30% shift in circle size.

3.2. Friedman ANOVA and Wilcoxon matched paired tests

The Kolmogorov-Smirnov test was used to examine the conformity of the obtained data with the normal distribution (Table 4), in order to select an appropriate statistical test for comparing the strength of the Ebbinghaus illusion across different samples.

It was determined that the data on none of the samples conformed to a normal distribution (Table 4). In Sample 1, where all circles are white, the empirical p-value is $p < 0.01$, as well as in Sample 3, where the inner circles are black and the outer circles are white (Table 4). In these samples, the Max D statistics are Max D = 0.37 and Max D = 0.36, respectively (Table 4). Sample 2 also did not conform to a normal distribution, as the Kolmogorov-Smirnov test showed a test statistic of Max D = 0.26 with an empirical p-value of $p < 0.05$ (Table 4).

Table 4. Kolmogorov-Smirnov test of the Ebbinghaus illusion results.

SAMPLES	Tests of Normality	
	Max D	**K-S p**
Sample 1: White circles	0.37	p < 0.01
Sample 2: Black circles	0.26	p < 0.05
Sample 3: White outer and black inner circles	0.36	p < 0.01

Legend: *The table contains the Max D statistic and the empirical p-value of the Kolmogorov-Smirnov test.*

It was determined that the data on none of the samples conformed to a normal distribution (Table 4). In Sample 1, where all circles are white, the empirical p-value is $p < 0.01$, as well as in Sample 3, where the inner circles are black and the outer circles are white (Table 4). In these samples, the Max D statistics are Max D = 0.37 and Max D = 0.36, respectively (Table 4). Sample 2 also did not conform to a normal distribution, as the Kolmogorov-Smirnov test showed a test statistic of Max D = 0.26 with an empirical p-value of $p < 0.05$ (Table 4).

For testing the existence of statistically significant differences among pairs of samples, a nonparametric Friedman ANOVA was selected. The results of this statistical analysis showed that there are pairs of samples that differ statistically significantly. The ANOVA chi-square value is Chi Sqr. = 7.85 with N = 27 and degrees of freedom df = 2. The empirical p-value is $p = 0.020$, which confirms the existence of pairs that differ statistically significantly.

To identify which pairs show statistically significant differences, Wilcoxon matched pairs tests were chosen for post-hoc analysis (Table 5). In the following Table 5, the empirical p-values of the Wilcoxon tests for the sample pairs are presented.

Table 5. Wilcoxon tests of the Ebbinghaus illusion.

SAMPLES	Sample 1: White circles	Sample 2: Black circles
Sample 2: Black circles	T=0.0	-
	p=0.012	
Sample 3: White outer and black inner circles	T=8.0	T=13.5
	p=0.090	p=0.154

Legend: *The table contains the T-statistic values and p-values of the Wilcoxon tests.*

The Wilcoxon test showed a statistically significant difference between Sample 1 and Sample 2, as the T-statistic is T = 0.0 with a p-value of $p = 0.012$ (Table 5). Therefore, the Ebbinghaus illusion is of greater intensity in the sample with black inner and outer circles. There are no statistically significant differences between Sample 1 and Sample 3 (T = 8.0, $p = 0.090$), with white circles and inner black and outer white circles. Similarly, there are no statistically significant differences between Sample 2 and Sample 3 (T = 13.5, $p = 0.154$), with black circles and the sample with inner black and outer white circles.

The results of the Friedman ANOVA and Wilcoxon tests clearly show that the colouring of the circles has a statistically significant effect on the Ebbinghaus illusion.

4. Conclusions

The Ebbinghaus illusion frequently occurs in printing when a smaller object is surrounded by larger objects, and vice versa, when a larger object is surrounded by smaller objects. In this case, the illusion acts by increasing or decreasing the perception of the central objects in relation to their actual physical size. The surfaces, i.e., the circles, can be coloured or uncoloured (black). In this study, it was statistically proven that colouring the surfaces black has a statistically significant effect on the strength of the achromatic Ebbinghaus illusion. This conclusion was confirmed by the Friedman ANOVA, whose results gave Chi Sqr. = 7.85 with N = 27 and degrees of freedom df = 2, with an empirical p-value of p = 0.020. Post hoc Wilcoxon tests confirmed that there is a statistically significant difference between Samples 1 and 2 (T = 0,0, p = 0.012), which means that the illusion is stronger when all circles are coloured black (Mean = 18.15%, Med = 20%) compared to the situation when the circles are not coloured (Mean = 15.93%, Med = 10%). However, it should be emphasized that no statistically significant differences were found between Sample 2, with all circles coloured black, and Sample 3, with white outer and black inner circles (T = 13.5, p = 0.154). Although the descriptive statistical parameters indicated a stronger illusion effect in Sample 2 (Mean = 18.15%, Med = 20%) compared to Sample 3 (Mean = 15.93%, Med = 10%).

Therefore, in printing, attention should be paid to areas where the illusion occurs, especially in situations where it appears on black-coloured surfaces, in order not to compromise the quality of printed graphics. Further research should focus on using different colours for the Ebbinghaus circles, to examine the intensity of the optical illusion on chromatic samples in printed media, with the goal of more effective management of the printing process.

Conflict of Interest: The authors declare no conflict of interest.

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